Estimates for derivatives of the Burgers equations in terms of viscosity

Andrei Biryuk

Consider the one dimensional generalised Burgers equation

$$u_t + f(u)_x = \delta u_{xx} \tag{1}$$

with a ℓ-periodic and C^∞-smooth initial state. The function f is also C^∞-smooth and strictly convex ($f'' \geqslant \sigma > 0$). Equation (1) is studied in the small dissipation regime i.e. δ is a small positive parameter ($0 < \delta \ll 1$).

We say that quantity g_δ has order δ^{-s} if there exist wo δ independed constants c and C such that for $\delta \in (0,1)$ we have: $c\delta^{-s} \leqslant g_\delta \leqslant C\delta^{-s}$.

Take time $T = \frac{2\ell^{3/2}}{\sigma|u_0 - \langle u_0\rangle|_{L_2}}$. Here $\langle u_0\rangle$ is the mean value of the initial state u_0.

It turns out that averaging of C^m-norm of solution over $[0,T]$ has order δ^{-m} and, for $m \geqslant 1$, the corresponding averaging of the Sobolev H^m-norm has order $\delta^{-m+1/2}$. Using explicit expression for a Sobolev H^m-norm via Fourier coefficients we can get some information for the behaviour of the Fourier coefficients as $\delta \to 0$.