Publications list for Andrei Biryuk

[1] Biryuk A. *Lower bounds for derivatives of solutions for NLS.* Submitted to "Proc. Royal Soc. Edinburgh Sect. A".

[2] Biryuk A. *On Invariant Measures of the 2D Euler Equation.* J. Stat. Physics. Vol 122, no 4, (2006), pp. 597-616

[3] Biryuk A., Craig W., Panferov V. *Strong solutions of the Boltzmann equation in one spatial dimension.* C. R. Acad. Sci. Paris, Ser I, Vol. 342, no. 11 (2006) pp. 843-848.

[4] Biryuk A., Craig W., Ibrahim S. *Construction of suitable weak solutions of the Navier-Stokes equation.* Contemporary Mathematics – AMS, 2006 (to appear, page volume: 18 pages)

[5] Biryuk A., Craig W., Panferov V. *Smoothness of solutions for the Boltzmann equation.* (in preparation, approximate page volume: 60 pages)

[6] Biryuk A., Craig W., Ibrahim S. *Questions related to the uniqueness problem for weak solutions of the Navier-Stokes system.* (in preparation, current page volume: 12 pages)

[7] Biryuk A., Kuksin S. *Remarks on the Navier-Stokes Equation.* (in preparation, current page volume: 44 pages).

[8] Biryuk A. *About marginal density distribution.* Preprint 2005 (Russian. English summary).

[9] Biryuk A. *On Multidimensional Burgers Type Equations with Small Viscosity.*, Contributions to Current Challenges in Mathematical Fluid Mechanics. Series: Advances in Mathematical Fluid Mechanics, Galdi, Giovanni P.; Heywood, John G.; Rannacher, Rolf (Eds.) 2004, pp. 1-30. Birkhauser, Berlin.

[10] Biryuk A. *Note on the transformation that reduces the Burgers equation to the heat equation.* Preprint: mp_arc 03-370. (2003)

[11] Biryuk A. *Distribution of the component of the uniform distribution in a polyhedron.* Preprint 2003 (in Russian).

[12] Biryuk A. *On Spatial Derivatives of Solutions of the Navier-Stokes Equation with Small Viscosity.* "Uspekhi Mat. Nauk", Vol.57 No 1., (2002). English translation in "Russian Math. Surveys" **57** (2002), no. 1.

[13] Biryuk A. *Spectral Properties of Solutions of Burgers Equation with Small Dissipation.* Functional Analysis and Its Applications. Vol. 35., No 1., (2001), pp. 1-15.

[14] Biryuk A. *On generalized equations of Burgers type with small viscosity.* Proc. Int. Conf. "Differential Equations and Related Topics", Moscow–2001.

[15] Biryuk A. *Spectral Properties of Solutions of the Generalized Burgers Equation.* "BAMC-2000", Manchester–2000.

[16] Biryuk A. *Estimates for derivatives of the Burgers equations in terms of viscosity.*, Abstr. Tagungsbericht "Analytical and Statistical Approaches to Fluid Models", Oberwolfach–2000.

Brief reviews

In [1] we improve the previously known lower estimates for the derivatives of solutions of the nonlinear Schrödinger equation in the turbulent regime (when the coefficient near the Laplacian term is small).

In [2] we give a rigorous proof for a physical conjecture that two important classes of invariant measures for the 2D Euler equation are indeed different. This fact was known before only on a heuristic level. We study further properties of these complicated classes.

In [3],[5] we study the Boltzmann equation, which describes the evolution of the mass probability density function $f(x, v, t)$ for particles distribution in the position-velocity phase space. A new averaging property of the collision operator and new integral inequalities allow us to develop new methods of constructing global strong solutions. In particular, we develop new estimates that allow us to control the L^∞-norm of the solutions. Through a weak/strong uniqueness principle due to P. -L. Lions, these solutions are unique among the class of dissipative solutions. Furthermore, we show that the Boltzmann flow propagates the moments and derivatives in both x and v coordinates. Currently we have completed results [3] for the case of narrow shock tube settings (or the slab geometry settings), which is to say that dependence on the spatial variable x is one dimensional. However the dependence on the velocity variable v is nevertheless fully n-dimensional, $n \geqslant 3$. In [5] we obtain new strong Lipschitz estimates for nonlinear Boltzmann collision (integral) operator. The novelty is that, unlike the other authors, we consider the space inhomogeneous case. Thus we can treat the case $\dim x > 1$.

In [4] we construct a suitable[1] weak solution for the Navier-Stokes system on a torus $\mathbb{T}^3$ via a Galerkin-superviscosity procedure. For the case of a torus $\mathbb{T}^d$ of an arbitrary dimension d, we obtain new uniform upper estimates on the Fourier coefficients of the solution (Theorem 5.3).

As a corollary to [6] we have the following result: if space-time singular sets of two weak solutions are disjoint, then both sets are empty, i.e., both solutions are smooth and must coincide.

The review paper [7] is devoted to describing the present state of the art of the theory for the Navier-Stokes system.

In the sequence of preprints [8, 11] we study uniform probability distributions in polyhedrons of large dimension and their marginal properties. Easily formulated, this problem is much more complicated than it seems. Some direct application can be derived in the field of numerical analysis and statistical physics.

In paper [9] we establish upper and lower bounds for the derivatives of space-periodic solutions of multidimensional Burgers–type equations with an arbitrary flow-type non-linearity. The number of spatial dimensions (n) may differ from the number of unknown functions (m). Establishing the lower bounds is the main aim of paper [9]. The estimates for the lower bounds given in [9] hold, provided that the initial condition satisfies certain non-degeneracy assumptions. In the 2D case ($m = n = 2$) we show that the non-degeneracy is a necessary and sufficient condition for the lower bounds to hold.

In [10] we prove a uniqueness theorem for the viscous Burgers equation without any growth restrictions. We remark that for the linear heat equation one would need to impose the Tikhonov growth restrictions for the uniqueness result.

In [12] we obtain lower bounds for derivatives of space-periodic solutions of the Navier-Stokes equations with a non-degenerate initial state. In the 2D case the degeneracy is equivalent to the absence of pressure.

Paper [13] and the proceedings articles [15], [16], are devoted to the 1D generalized Burgers Equation with small viscosity. For derivatives of its solutions we obtain upper and lower bounds, which are optimal in terms of powers of the viscosity. This allows us to formulate the Kolmogorov–type spectral law for the solutions and to calculate the corresponding Kolmogorov exponent.

The manuscripts are available at `http://www.math.mcmaster.ca/~abiryuk`

[1] This term stands for a special class of weak solution, for which a better regularity is known.